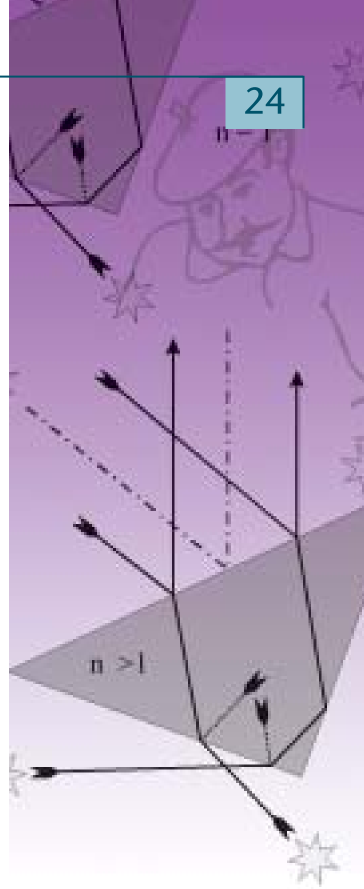

Optimizing faceting for beauty

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Abstract: *Demands for beauty of a gem are related to the level of cutting technology that has been attained. Working the stone unavoidably results in its loss of weight, so the beauty of a gem is in conflict with its potential value. The appearance of a faceted stone is produced by optical effects; therefore the problem of optimizing the faceting of a stone is reduced to determining the criteria which affect its beauty and defining the faceting parameters which best satisfy those criteria. A correctly faceted stone is a compromise between different criteria.*

This selection of optimum parameters is affected by the physical properties of the gem and how it will be used. A faceted gem is an optical device which transforms light sources into light specks on the surface, which is different for each position of a viewer. The stone will be set in a jewellery article which will most probably be seen from certain directions, thus its appearance should be optimized for the most likely combinations of viewer and light source positions. For example, for a viewer looking perpendicularly into the table, one can determine the directions of light sources which are visible in the stone. In optimizing the directions of light sources it is important to consider the influence of the viewer's head and body, which are not light sources. The body of the observer can lead to asymmetrical distribution of light on the surface of the stone and incorrect facet slopes can lead to 'dead' regions of the gem.

To optimize the arrangement of facets an artist will consider the appearance of the entire gem.

Translator's Note

It is impossible to translate exactly. Word-for-word does not read properly and re-phrasing requires some word changes. In technical writings many terms have unique meaning in the subject field and are different from those in dictionaries or not listed at all; in such cases the translator must interpret the best meaning from context. It is necessary to know the subject and, ideally, the author and his intent. In this case we were fortunate to talk often with the author via Internet to clarify meanings, and ultimately, to have him review the translation.

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The history of faceting

The desire for beauty has been a characteristic of mankind throughout his history. This is shown by cave paintings and archeological excavations. Natural stones have played an enormous role in evolution of society; note the term 'Stone Age'. However, the ways of using minerals for adornment, their variety and popularity, have depended on methods of working them at the time.

The earliest stones used for adornment were probably water-worn alluvial pebbles. Later, for greater lustre, their surfaces were polished with abrasives. Because men were unable to sort abrasives by grain size, final polishing was probably done with a soft abrasive-carrier such as leather or wood. It was impossible to achieve a perfect flat surface but convex surfaces were relatively easy to polish. The hardness of accessible abrasives did not exceed that of alumina (Al_2O_3), which limited the variety of workable materials. Such materials were not valued then as they are today. Archeological discoveries and etymological studies of the names of precious stones[1, 2] show that before mastery of fine polishing techniques was common, only vividly coloured opaque minerals were popular – lazurite, turquoise, jasper, nephrite, jadeite, coloured chalcedony, etc. It was easy to approach fine polishing of the surfaces of most cryptocrystalline minerals in this list. Materials such as ruby, sapphire or emerald, and especially diamond, probably were not used then.

The next step must have been the flattening of one side of a pebble to facilitate its attachment to the surface of everyday objects, weapons or adornments. Thus arose an early form of gem-cutting called the 'cabochon'. The skills of working different stones improved considerably: carving appeared on hematite and chalcedony, and work began on transparent stones. At this time diamond was still not valued or used in adornments. Although diamond is sometimes found in nature as a well-formed octahedral crystal, the configuration of its natural faces, in combination with its optical properties, means that it does not effectively show its 'fire' (colour flashes) so valued in modern diamond cuts. Stones with convex surfaces, however, showed the phenomena caused by oriented inclusions such as 'asterism' and 'chatoyancy' very well. Stones with these effects were most highly-valued until the development of faceting with flat faces. The values of such stones were uncommonly high in regions where the quality of faceting remained poor until recently.

You need only to turn over a transparent cabochon – so that the base faces upward – to see that it looks better, that it reflects more light into the eye of the observer. Note the similarity of the form of faceted stones from regions with less advanced faceting with the form of the inverted cabochon. The evolution of cutting by flat faces is entirely natural - from the cutting of cabochons the cutters learned that a perfectly smooth rounded surface was harder to obtain than the interrupted surface formed by flat faces of arbitrary form. To form truly flat faces it is necessary to use a flat metallic disc (copper, tin or lead), like that used by a potter, for grinding and/or polishing. Initially the arrangement of facets was random, determined largely by the shape of the rough. The criterion of quality was the beauty of the external view of the gem. Cutters noticed that some stones looked better than others and tried to repeat the special features of the more beautiful ones. Thus evolved, by trial-and-error, good (but certainly not the best) proportions for faceting3. This was before the first attempt to mathematically define these forms. The irregular shapes of crystal fragments and alluvial pebbles did not suggest symmetrical arrangement of the facets; exceptions were the elongated crystals of beryl and tourmaline, the shape of which became the basis of the baguette and emerald cuts.

Generally the evolution of faceting follows the example of diamond[4]. Actually, when man first thought to polish the top of the diamond octahedron, the faceting of other minerals was already well-known and the proportions of these stones sufficiently perfected. Fortunately, the cubic face is the softest for diamond. The slopes of the octahedron faces were so imperfect that it was necessary to polish a facet at the opposite end – the culet[5] – of significant size, but still the stone did not look good. Therefore cutters began to change the slopes of all the faces and, in embellishing the stone, added additional bevels with reduction and gradual disappearance of the culet. The resulting arrangement of facets is due to the four-fold symmetry of the initial octahedron. Thus was born the modern arrangement of diamond facets, with the shape of the girdle being defined by the square form inherited from its original octahedron. This style of cutting is now called 'antique' or 'cushion'. Only the tendency to form the diamond like other faceted stones explains the cutting-off of the top, for one, and subsequent efforts to alter the natural octahedron. The start of its faceting was the key to the destiny of diamond; as a result diamond acquired exceptional popularity and high price universally. As cutting experience was gained, the proportions of the gem changed and its beauty increased. In the course of time, beauty of the gem gained greater importance than its weight whereby, in modern cutting, the girdle of the most popular diamond cut, the standard round brilliant (SRB) has become perfectly circular. Here, for the first time, we see conflict between the weight and beauty of the gem; this subject will be pursued in detail later. The first attempts to calculate the parameters of faceting were made in the 18th century[6, 7], and best-known in our time came the book of Marcel Tolkowsky in 1919[8, 9]. Despite some errors[9] it was an attempt to mathematically explain the successful solution which had, by that time, already been found by the best cutters in the trade. The wide reputation of this book explains its timeliness – it appeared when the public was ready to consider that diamond should be cut only with precisely correct proportions. However, the main merit of this book is that it drew attention of a wider audience to the problem of correct parameters for faceting and served as the source of ideas for a series of other studies[10, 12]. Tolkowsky's SRB solution became the basis of several standards, including some modern systems for appraising the cut-quality of diamonds. Similar studies were also conducted in Russia[13, 14].

Beauty and value or value vs. beauty

The faceted gem may serve one of two main purposes. On the one hand, the gem is the embodiment of beauty and part of jewellery, whose primary purpose is to please the eye. On the other hand, there is its material value as an investment, when it is, perhaps, kept in a bank. Strictly speaking, for the second purpose the stone can exist by itself and need not be seen – there are only important documents confirming its existence and indicating its characteristics which determine its value. Very frequently for trading (especially wholesale) there is the need for commercial transactions without seeing the stones or by people incapable of determining their quality. For this there are special systems to define and evaluate the qualitative and quantitative parameters of the stone. In modern systems the quality of cutting is appraised by separate parameters. A specialist-expert appraises the stone according to one of these systems (certification) and issues a correspondingly designed form (certificate). The illusion can result that the criteria by which the stone is certified completely represents its beauty and desirability to a consumer, which is generally not true. Certainly they try to select grading system parameters so that they do not conflict with the consumer's perception of the stone, but complete agreement is impossible to attain. The main problem of any system of certification is to provide simple and universal guidelines as to how to rank the stones. In this sense it is not required that the certification criteria fully characterize beauty, which is a very complex property of the gem and depends upon too many environmental factors including our subjective perception. The beauty of a stone and its cost are two aspects of the same thing; we should not blindly subjugate beauty to cost, or to an appraisal system unless the stone is cut exclusively for investment purposes. More than any other mineral, the beauty of diamond suffers from this double standard. From here on we will consider only questions related to achieving beauty, not looking for the best compromise of beauty and cost, but developing each cut according to artistic taste, preference, experience and objectives.

Let's now list the four cases where existing systems of appraisal and trade operate at the expense of beauty.

1. Shape vs. weight

The sum total of beauty, rarity and durability determines the attractiveness of a stone and its price. Demand for beauty of a cut stone has changed according to changes in technology of cutting and the results which it made attainable. For people outside the gem cutting industry the beauty of gems was determined only by their colour and size (weight). Since the beginning of gem-cutting, form has become a large factor, unavoidably accompanied by its loss of weight. Form began to play an especially important role in the faceting of transparent gems. Historically, in regions where the technology of faceting lagged, the main consideration was conservation of the stone's weight. However, in countries with developed faceting technology, the priority is the correct shape of the faceted gem. Thus the majority of Indo-Chinese and Indian cutters do not favour losing weight when cutting rubies and sapphires like diamonds and European and American jewellers have a problem mounting stones in jewellery which are faceted in this way from Southeast Asia. We expect that the correct shape of the stone and its resulting beauty will improve in the course of time.

2. Weight vs. size

It is understandable that a dimensionally larger stone pleases the eye more than a small one. But people have become accustomed to buying goods by weight; they typically do this both with potatoes and with precious stones! However, the weight of a stone is not always proportional to its visible size. Round stones faceted in a country with developed faceting technology, having the same girdle diameter (that is, the same visible size) will be lighter but more beautiful than some faceted in southeast Asia. Unfortunately some cutters cut a gem worrying only about retention of its weight, not taking its beauty into consideration. The consumer suffers. Judge for yourself – you get a heavier stone for more money, but less beautiful. For less beauty the buyer is forced to pay more money! This approach is most common in those countries where the cutters do not know how to cut or cut badly; they sell them as a certain substance by weight (they can pour more than 100 stones into one bag); in such cases the price of the finished stone differs little from the cost of the raw material from which it is made. This is like appraising a painting by the cost of its paint and canvas (this approach is justified for some paintings!). Thus stones sell by weight, not that weight is their best characteristic, but because weight is a simpler quantity to measure, and is the tradition.

3. Discrete price lists

To price stones the trade creates price lists, which are sometimes complex tables in which, as a collection of charts, the parameters of stones are divided into discrete intervals. The price of a stone jumps when it crosses an interval boundary. For example, on all price lists the price of a stone weighing 0.99 carat is considerably less that a stone weighing 1.01 carat. If the faceting of these stones is identical they cannot be visually distinguished from each other. Cutters, concerned with profit, cut stones to dimensions which correspond approximately to carat sizes, but to obtain the larger profit will deviate from the best proportions and, to be sure not to miss, will try to add a little extra; thus the beauty of the stone may suffer considerably. Similar step-grading is also used for other parameters of the stone.

4. Restriction of shape and other parameters

Systems for appraising the quality of cutting for diamonds use the standard set by

Tolkowsky which, although good, is not best for specific jewellery articles. Its very specific arrangement of facets is accepted as the standard, therefore the faceting of a more beautiful stone with small deviations from these parameters can cause a reduction in its price. Non-standard faceting, even if more beautiful, is often met with hostility by stone dealers although the jeweller, to implement his design ideas, may need gems of non-standard shape and facet arrangement.

Two approaches to improving the form of faceted gems

Faceted gems involve rather complex optics. Two fundamentally different approaches are possible for the solution of any complex problem.

1. *General* (also called 'synthetic') consists of examining the problem as a whole. This method applies to improving the appearance of a faceted gem. A cutter, after completing his work on the stone, can evaluate the result by critically examining the item; it is possible to determine the best combination by comparing stones faceted with different parameters. So, by trial-and-error, experimenting cutters searched for their favourite solutions. This method is good in that it uses the exact 'image' of the faceted gem! However, this approach involves expenditure of raw material and time, although, under conditions of production manufacture this can be minimized by controlled variations in faceting parameters of individual stones. This is exactly how M. Tolkowsky found the solution which he tried to verify in his book[8]. Appearance is subjective but we can photograph a real stone in real lighting, and quantitatively evaluate its real image.

Only recently, with the advent and wide acceptance of computers, did it become possible to mathematically simulate a faceted stone with sufficient accuracy15-18, to draw and analyse its image by program17, 18. Speed and elimination of raw material loss are the advantages of this method. It is possible to criticize many possible re-reflections of rays (to discuss the finer points) taking into account partial polarization of refracted beams, and similar small deficiencies, but we must acknowledge that the best of such programs provide quality representations and make it possible to draw valuable practical conclusions18. The results are very entertaining and produce an especially strong impression on unenlightened people. However, the computer image, at best, only approximates an appearance of a real gem; therefore, with this method, in principle, it is not possible to gain more information than by cutting and studying real stones. Most important – a single external view does not give direct answers to numerous questions, such as: why this stone, under certain conditions behaves one way and not another. Having received answers to such questions, it is possible to learn how to design a faceted gem.

2. An *analytical* approach consists of separating the total problem into less complex components and examining them separately; for example, analysis of rays incident to the stone from a specific chosen direction, or a single row of facets, or only in one plane. It is possible to study only the influence of table size on the appearance of the stone or separately calculate only the ability of the gem to return light to the eye of the viewer. Until now the majority of researchers have adopted this approach and 'could not see the forest because of the trees'. Usually one researcher would focus on one method of solving only one part of the problem without considering other aspects. Even precise answers to the question – why does this facet work precisely so, in this condition, and not otherwise? – does not make it possible to estimate the beauty of the entire stone.

In reality these approaches are never encountered in such pure form. Even those who attempt to evaluate appearance by the method of computer simulation, divide the problem into smaller parts; for example, they

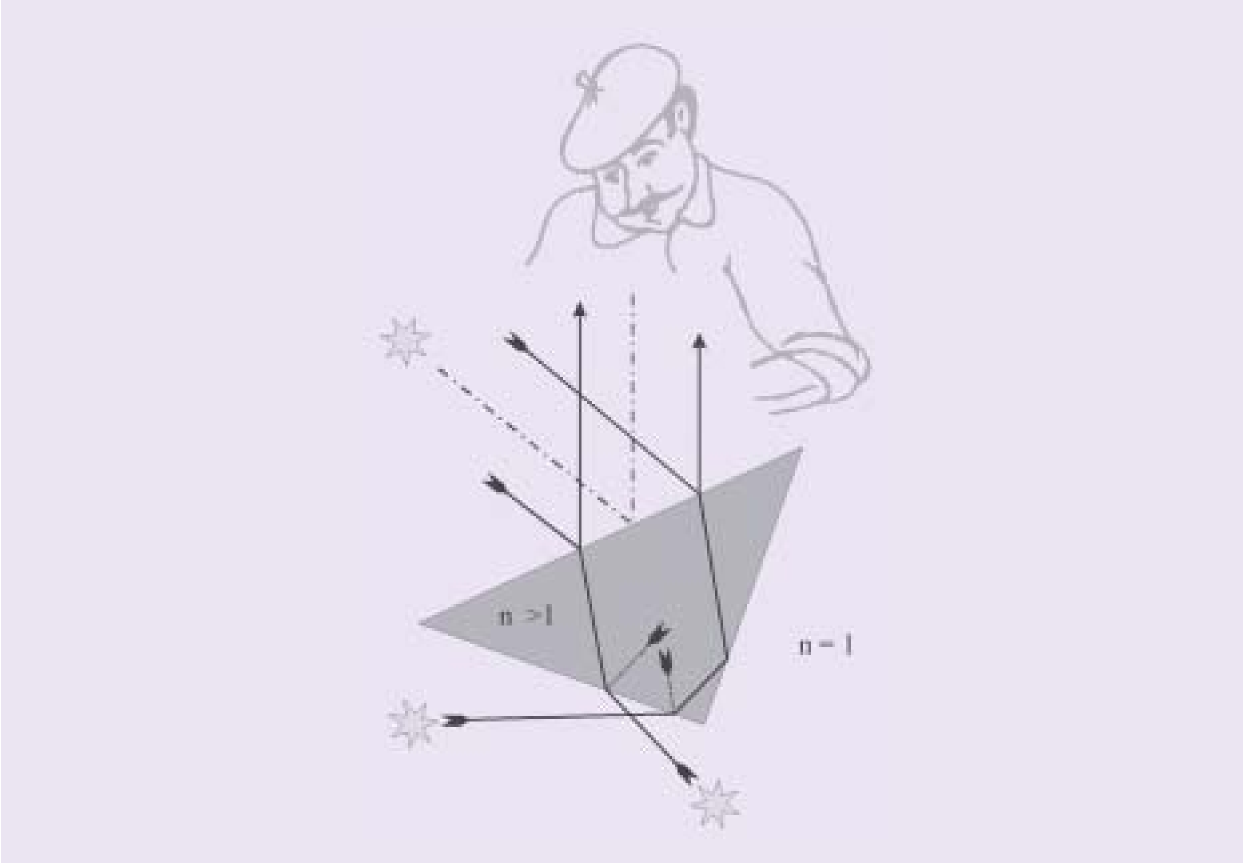

Figure 1: *Light rays in a faceted gem of ar-bitrary shape*

separately evaluate light return and dispersion colours of rays. Even here there is unavoidable simplification – limitation to a number of re-reflections of rays, or the number of colours in the spectrum, etc. Those who choose the analytical approach must, in the end, combine the solutions of all the components of the problem. The best results in this case are obtained by those researchers who are intimate with all aspects of gems from physics and mathematics to the cutting and manufacture of finished products (Marcel Tolkowsky, Bruce Harding, Anton Vasiliev*).

From here we will attempt to find how to optimize the form of a faceted gem to achieve its maximum beauty by using both approaches. One must treat the results of such studies not as the final truth, but only as tools, similar to the brushes and paints of the artist. The artist is required to create beauty, and he can use any combination of brushes and paints.

*Vasiliev added by translator

What we see in faceted stones

The appearance of a faceted stone, despite the subjectivity of the concept of beauty, is defined by optical effects. Therefore the problem of optimizing the stone by its facets is reduced to determining criteria on which its beauty depends, and to selecting faceting parameters (shape, facet slopes, and their arrangement) to best satisfy the selected criteria.

First of all, let us list what we generally see in a faceted gem, which is a convex polyhedron of arbitrary form, bounded by flat surfaces. First, consider only stones in air, of faceted, colourless, optically isotropic material with a refractive index more than one. After defining the position of the stone and the observer (*Figure 1*) we can then trace the path of each ray that leaves the stone and enters the pupil of the viewer's eye. For such rays to exist the following three factors are necessary:

1. source or sources of light;
2. the stone itself;
3. the viewer.

Usually the distance from the eye to the stone is much more than the size of any facet, the distance to the light source is still more, and the width of the light source exceeds any facet of the gem. What the viewer sees in each part of the stone can be defined by extending all rays from the eye to the gem to their intersection with surrounding objects (*Figure 1* shows these as stars). If, in this direction, there is an object radiating light, we will see a speck of bright light in the corresponding place on the stone. If a dark object is encountered, the corresponding spot on the stone will be dark. Since the eye of the viewer is focused on the stone, a sharp picture is focused on the retina (the rear focal plane of the eye's lens) which is a mosaic of these light and dark spots. Each spot corresponds to one (or more) objects surrounding the stone. Consequently we see in the stone only reflected and refracted images of the surrounding space. Directions (relative to stone) to objects visible in the stone can be assigned corresponding angles, for example by azimuth and inclination. *Thus the faceted stone is an optical device which converts a distribution of light sources into the pattern of light specks seen on its surface*. The geometry of the faceted stone defines the manner of this conversion.

The more bright specks of light we see on the surface of the stone, the stronger it will shine. Let's call this quantitative characteristic the 'brightness' of the stone (the concept of 'brightness' is used here in the everyday sense, not in the physics sense). We can optimize the stone's brightness: if lights are placed in the same directions as each ray, the entire surface of the stone will shine! If they are moved the stone will become dark. Thus the degree of brightness of the stone depends not on its form but on the arrangement of the light sources. It depends on the form of the stone only where the light sources must be in specific places. We obtain the paradoxical conclusion that there is no poor faceting, only unsuccessful arrangement of the light sources!

It is a simple fact that the task to optimize faceting of a stone to increase its brightness for the most general case, does not make sense. There cannot always be 'ideal' cutting; it is possible to optimize one faceting parameter or another to achieve greater beauty only for given specific conditions. Now let us put limits on relative positions of light, stone and viewer.

Light return

A real faceted stone is an adornment, more precisely part of an adornment because it is rarely used for this purpose in loose form. Therefore we will study the faceted stone fastened to the surface of jewellery or other article. Let us represent this surface with a plane which passes through the stone, dividing all surrounding space into two halves. Momentarily, let us call these halves the top and bottom so that the major part of the jewellery article is found in the lower half and the viewer and light sources in the upper. Since the light source and viewer are found in the same half of space, to return light rays to the viewer the stone must *reflect* them. Simple refraction is usually insufficient to make them hit the eye of the viewer.

The polished surface of any crystal will shine by directly reflecting light. This Fresnel partial reflection does not depend on the shape of the stone (the reflected beam does not enter the stone) – it is determined by the refractive index (and angle of reflection) – therefore there is no sense in trying to optimize this lustre. Total reflection can be ensured by spraying on a metallic reflecting film, an amalgam coat, or a simple layer of foil under the stone; however, all such reflective layers are destroyed in the course of time, violating a criterion of treasure – its eternity. In most modern types of faceting the reflection of light into the eye of the observer

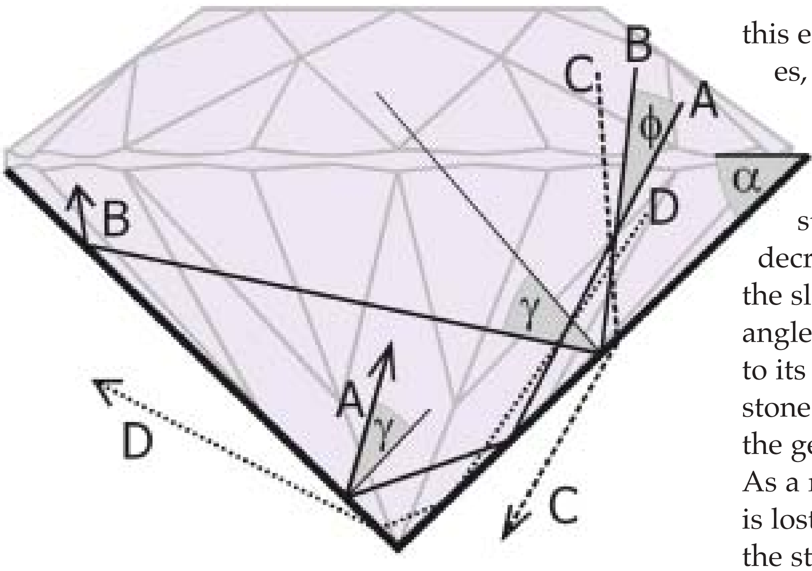

Figure 2: *Internal reflections from pavilion facets.*

is achieved by double total internal reflection by the facets of the pavilion. We will examine other methods of recovering light in future publications.

Let us trace the path of a ray, shown in *Figure 2*, via two opposing pavilion facets inclined at angle α from the plane of the girdle (the 'belt' of the gem). The conditions for total internal reflection of both facets are satisfied only for rays whose directions of incidence lie inside angle ϕ formed by limiting rays A-A and B-B. (Note that this range of directions ϕ inside the stone will be broader outside the stone because of refraction.) Total internal reflection is lost for ray C-C at the first facet and for ray D-D at the second. Violation of total internal reflection does not mean that the ray does not entirely cease to be reflected, but the portion of the beam which is reflected drops rapidly as the angle of incidence reduces below the critical angle γ. The range of directions for all incident beams which satisfy the condition for total internal reflection can be calculated:

$$\phi = 180° - 2\gamma - 2\alpha \quad (1)$$

where $\gamma = \arcsin(1/n)$ - the critical angle for total internal reflection for the given material with refractive index n. It follows from this expression that as α decreases, ϕ increases, and the more incident rays in the stone are turned back, the greater is the possibility of their being seen, with consequently greater brightness of the stone. However, this slope (α) cannot be decreased below a certain limit. If we make the slope of these faces less than the critical angle, with the stone viewed perpendicular to its table, everything located below the stone is visible through it and all regions of the gem under the table become transparent. As a result, perception of the stone's solidity is lost and it looks 'like glass'. When tilting the stone one or more faces of the pavilion may become transparent, (especially with low refractive index) but if condition $\alpha > \gamma$ is met, only a small part of the total number are transparent, which is not as noticeable as transparency of the entire middle. In the opinion of experimental faceters[23] slope α must exceed the critical angle by 1.5-2 degrees:

$$a > g + 2° \quad (2).$$

The direction of the view of the observer perpendicular to the table is certainly important but not the only view possible. The priority of viewing directions for studying a stone depends on its application. The owner of a ring will most likely examine the stone perpendicular to its table. Stones in earrings are viewed from various directions, although mainly from the front (i.e. those parts not enclosed by the mount). Notice that everyone except the wearer sees a ring stone from abitrary positions that the wearer does not.

Figure 2 shows the advantages of open-sided settings for holding gems; since parts of the casting leave the pavilion accessible to ambient light, rays D-D cannot be considered lost. This is especially advantageous for colourless stones since the absorption of such rays is less (their path through the stone is shorter, therefore they are lighter) and dispersion – that is, caused by the dispersivity of the material of the gem – is sometimes higher than usual. Consideration of rays D-D demonstrates that the optimum form of faceting depends also on how the stone is held in the jewellery.

In order to see a ray reflected by the stone, it is necessary not only to return it via the pavilion, but also to ensure its exit to air through the top of the gem. If the slope β of the crown facets is too great, instead of leaving the gem, the ray will be reflected back into it, as shown in *Figure 3*. Although

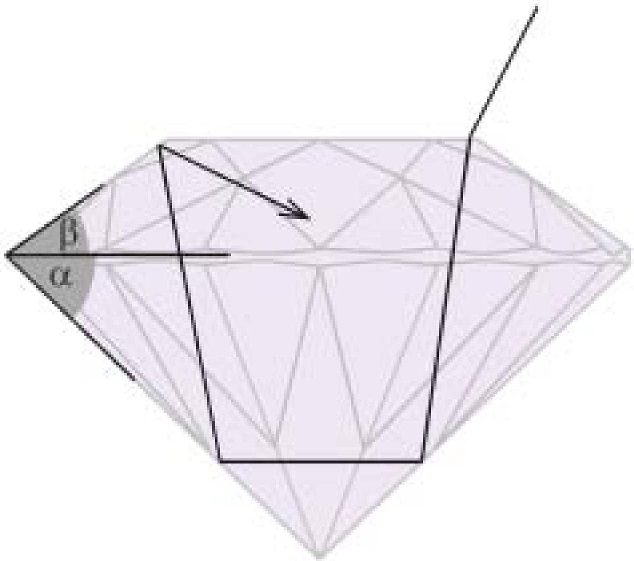

Figure 3: *Total reflection of ray from crown back into gem.*

faceting methods were proposed, by calculations for repeated internal reflections (6 and more times), it is unlikely that the authors actually tried this in practice; the effects of imperfect polishing, inaccurate geometry and un-flatness of facets in real gems accumulate with an increase in the number of reflections, and the path of a ray becomes unpredictable. It is possible to avoid internal reflection by the crown facets if we cut them at slope angle β not exceeding ...

$$\beta < \gamma + 90° - 2\alpha \text{ (3).}$$

Effect of the viewer

The presence of a viewer puts additional limits on arrangements of light sources. The fact is that the head and body of the viewer are not light sources (at least not bright). The head is in the same direction as the eye but has a significant angular size (usually the light source is farther from the stone than the viewer), it is desirable to facet so that the viewer will not see reflections of his own head in the stone. Areas of the gem which reflect the viewer's head will be dark for any positions of the light sources. The shape of the head and arrangement of the eyes vary for different people; shapes and sizes of hair-dos and headwear even more so. To simplify analysis, we consider the head as a sphere with an eye in the centre. Thus light rays exiting the stone must diverge from their direction of entry by more than half the angular size of the head. Ignorance of this fact lead to the design of Johnsen[10-12], which calculated, in essence, that the viewer would see only his reflection and only the lamp saw all of the light specks. To this day, researchers fall into this trap[21]. Divergence of the exiting ray from the entering ray must be assured for the three possible paths of the ray through the gem:

1. Entry through the table and exit from the same.
2. Entry through the table but exit from the bezel (side facets of the crown). Accord-ing to the principle of reverse ray-trac-ing, this is the same as entry into the bezel and exit through the table.
3. Entry through one side of the bezel and out the opposite side of the bezel.

Consideration of the viewer's head was first presented by Bruce Harding in an article[19] as early as 1975. He cut a garnet according to a published recommendation (40° slope of the main facets, top and bottom [3]). Upon examining it closely (by a lamp behind his head) Harding noticed that, as the stone neared his face (increasing the angular size of his head) it darkened, but it brightened as it was moved away from himself. He investigated this effect and published an article with analytical expressions for calculations, and graphs whose coordinates were the slope angles of the pavilion and bezel facets. He shaded those areas which did not provide

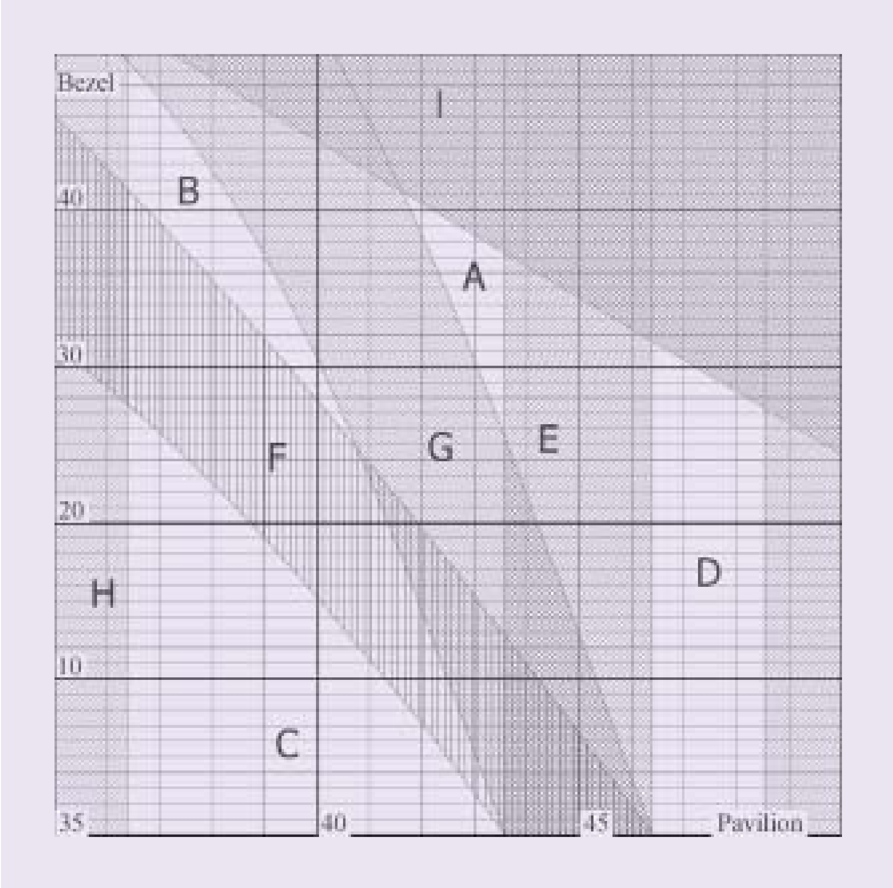

Figure 4: *Regions of pavilion and crown slope combinations.*

sufficient divergence of the exiting beam (similar to *Figure 4*), assuming that the viewer looks into the top of the stone perpendicular to the table. To illustrate this, Harding later (1986) wrote a computer program which showed the paths of rays through the stone. The author became aware of this work in the early 80s and expanded the results to include the case where the viewer looks at the stones from other directions[22].

Figure 4 shows shaded areas designated by letters **E**, **G** and **F** for faceted corundum, inside which facet slopes do not assure 10° ray divergence, for arbitrary positions of the viewer, for the three routes cited above, respectively. These areas are somewhat wider than those presented by Harding and more severely limit the acceptable regions, but they ensure the necessary beam divergence in those remaining regions for any inclination of the stone relative to the direction of viewing. Shaded areas **H** and **I** do not satisfy equations (2) and (3) respectively.

Four regions – **A**, **B**, **C** and **D** – remain unshaded. To compare them it is necessary to consider the body of the viewer, which is also reflected in the stone and blocks light. For stones in regions **C** and **D** the body of the viewer is reflected predominantly only in one half of the stone (near to the viewer for **D** and away for **C**). This one-sidedness seriously affects the beauty of the stone, so we exclude such slope combinations as unsuccessful.

The location and shape of the remaining acceptable regions (**A** and **B**) depend on the refractive index of the material being cut and the angular size of the viewer's head – that is, the distance from which the gem is viewed. The final results of calculation for different materials and 10° minimum ray divergence, are shown in *Figure 5*. Other details are considered less important and are omitted in this diagram. In this diagram zone **B** does not exist for quartz; actually it appears only for minerals with refractive index greater than 1.62. For materials with lower refractive index there is only one region of solutions (**A**), which disappears if **n** < 1.47. Apparently natural glass occupies last place in the list of minerals which can be faceted satisfactorily, and minerals such as fluorite and opal cannot satisfy the requirements we have defined.

Other things being equal, stones of region **B** have high brightness and give more rays, but those from region A have greater colour intensity. Without going into detail, let it be said that stones from the upper left region **A** have the best colour dispersion of exiting rays. Although calculations are carried out only for rays in the plane of the paper in *Figures 2* and *3*, the results apply generally also to oblique rays (lying in other planes), with the exception of rays exiting into the air through bezel facets [formula (3)]. To avoid loss of skew exit rays it is necessary to use either additional facets (upper triangles of the crown = 'stars') or to use crown main facet slopes 3-5° below the upper boundary of region **A**. Small deviations beyond the recommended limits do not significantly affect the beauty of the gem; they are permissible.

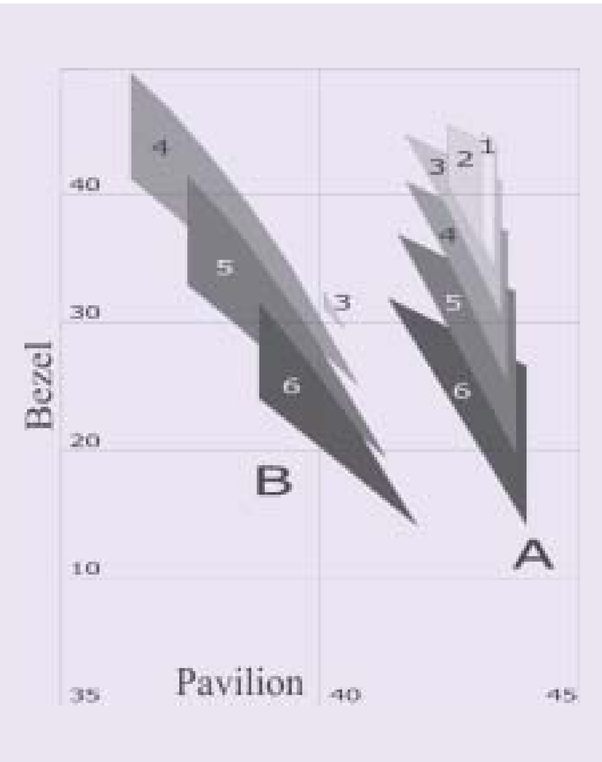

Figure 5: Best slope combinations for vari-ous gem materials
1) obsidian 2) quartz 3) topaz
4) corundum 5) zircon 6) diamond

In this article we examined only rays reflecting from facets which are opposite to each other. The majority of rays incident perpendicular to the girdle plane go exactly this way; however, other rays can enter adjacent or remote facets. In the latter case the path of a ray may change sharply and the majority of such rays pass through the stone by more complex routes, often with a large number of internal re-reflections. The number of such rays increases with tilting the gem; their main feature is a radical change in the mode and number of interacting facets by the tiniest rotation of the gem. The rays we studied comprise the majority and repeated part of all rays forming the exterior appearance of the gem. Therefore it is most important to consider them in the optimizing of faceting. If we seriously violate even one of the criteria set above, we will get a noticeable negative effect on the beauty of the stone. Precise calculation of all possible rays is possible but exceeds the scope of this article.

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